Running couplings in extra dimensions

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Abstract

The regularization scheme dependence of running couplings in extra compactified dimensions is discussed. We examine several regularization schemes explicitly in order to analyze the scheme dependence of the Kaluza-Klein threshold effects, which cause the power law running, in the case of the scalar theory in five dimensions with one dimension compactified. It is found that in 1-loop order, the net difference in the running of the coupling among the different schemes is reduced to be rather small after finite renormalization. An additional comment concerns the running couplings in the warped extra dimensions which are found to be regularization dependent above TeV scale.

1 RG in large extra dimensions

Recently the extra compactified dimensions have been attracting much attention as possibilities to explain various hierarchy problems; the gauge hierarchy, the Yukawa hierarchy and so on. The effective field theories in extra dimensions contain towers of massive Kaluza-Klein (KK) excitations, whose quantum effects alter the behavior of the running

[1]Parallel session talk presented at ICHEP 2000, July 27 - August 2, 2000, Osaka, Japan

couplings from logarithmic to power. Therefore the traditional picture of gauge coupling unification may be drastically changed [1]. In this talk we examine the running coulings explicitly in several schemes and consider the origin of power law running and their scheme (in)dependence [2].

In practice the notion of running couplings is not well defined in extra dimensions due to nonrenormalizability. We cannot help but defining the field theories by a certain cutoff. Then the Wilson RG is supposed to offer a natural framework to define the RG flows in such cases. There is another reason why the Wilson RG is suitable for the calculation of the β-functions in extra dimensions. The power law behavior of the running coupling is supposed to be generated by successive threshold corrections by tower of massive KK modes. The Wilson RG is faithful to their decoupling effects.

Therefore we apply the Exact RG, which is a continuum formulation of the Wilson RG, to a scalar field theory in $M_4 \times S^1$ space-time. In this formulation a certain cutoff is performed to the internal loop momenta, and the running coupling is defined by variation of the cutoff scale. We derive the β-functions for the four scalar coupling in the 1-loop level, assuming that the coupling is weak enough. We also calculate the β-functions in other schemes: the proper time regularization and the momentum subtraction, and compare these results.

In all schemes we obtain the β-functions as

$$\beta_\lambda = \frac{d\lambda}{dt} = b\epsilon_k(t)\lambda^2, \tag{1}$$

where scale parameter $t = \ln R\Lambda$ is introduced in terms of radius of the compact space R and the cutoff scale Λ. b is the 1-loop coefficient in four dimensions. The function $\epsilon_k(t)$ is dependent on each scheme k, and it's asymptotic form is given by

$$\begin{aligned} \epsilon_k(t) \quad &\rightarrow \quad 1 \qquad\quad \text{for} \quad t \ll 0, \\ &\rightarrow \quad B(k)e^t \quad \text{for} \quad t \gg 0, \end{aligned} \tag{2}$$

where B_k is a scheme dependent constant. This function is shown in Fig.1 by truncating the KK modes at N-th level. It is seen that the β-function shifts to the five dimensional

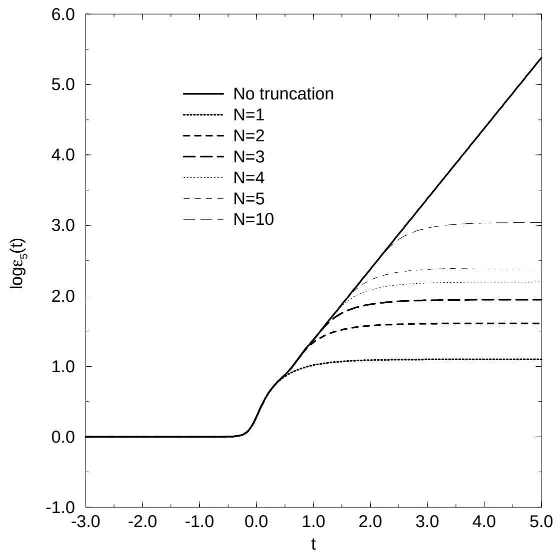

Figure 1: Scale dependence of the β-function coefficient calculated in an ERG scheme.

one smoothly by the KK threshold corrections. We may define the effective dimensions [2] by

$$D_{\text{eff}}(t) = 4 + \frac{d \ln \epsilon_k}{dt}, \tag{3}$$

which shows smooth transition from 4 to 5.

2 Scheme dependence

The β-functions given in Eq. (1) are scheme dependent even in the 1-loop level. However we may redefine the coupling constants in different schemes so that the β-functions match in the asymptotic region. The scheme dependence remained even after this procedure represents the net ambiguity in the RG in extra dimensions. The results obtained by explicit calculations are shown in Fig. 2. It is seen that the unremovable scheme dependence is rather small. In consequence, it is found that the GUT predictions for the low energy gauge couplings [1] are almost scheme independent [2].

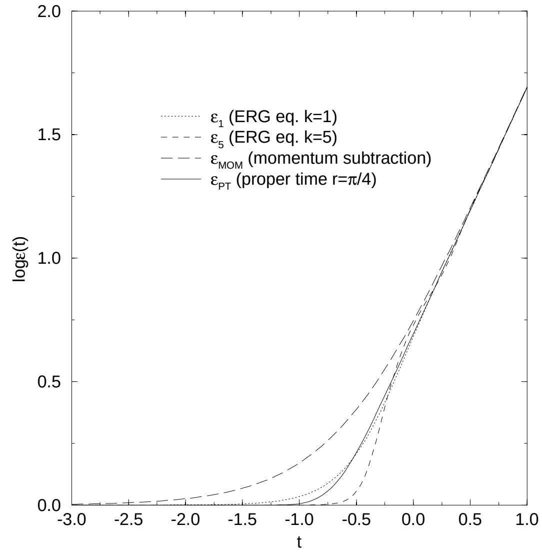

Figure 2: Scheme dependence of the net thershold corrections.

3 RG in the warped extra dimensions

The Randall-Sundrum type of compactification [3] is now under the most vigorous investigation. The extra dimension is bounded by two three branes, in one of which the Planck scale is reduced to TeV scale. A peculiarity of this compactification is that the mass scale of the KK tower appears at TeV order, even if the bare mass is Planck scale [4]. If we apply the RG scheme mentioned above to this case, the running couplings are found to show power law behavior too. This should be compared with the results obtained by the PV regularization [5]. However it should be said that the running coupling above TeV scale is totally regularization dependent, since the KK spectrum is influenced by the Planck scale (string) physics.

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